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B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2017
SECOND YEAR [BATCH 2016-19]
MATH FOR INDUSTRIAL CHEMISTRY [General]
Paper : III

Date : 22/12/2017
Time : 11 am – 2 pm
Full Marks : 75

[Use a separate Answer Book for each Group]

Group – A
(Answer any five questions) [5×5]

1. Find the shortest distance between the lines $\frac{x+2}{2}=\frac{y-1}{3}=\frac{z+1}{-1}$ and $\frac{x-1}{-1}=\frac{y+1}{2}=\frac{z-2}{4}$. 5

2. Find the point of intersection of the straight line $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+1}{-1}$ and the plane $2x+y+z=6$. 5

3. Show that the locus of the point of intersection of two tangents to the conic $\frac{l}{r}=1+e\cos\theta$ which are at right angles to each other is $r^2(e^2-1)-2elr\cos\theta+2l^2=0$. 5

4. Transform $x^2-6xy+y^2-4x-4y+12=0$ to canonical form. 5

5. Show that the distance of the point (3, 2, 1) from the line of intersection of the planes $x+y+z-4=0$ and $x-2y-z-4=0$ is $\sqrt{6}$ units. 5

6. Show that the polar of any point on the circle $x^2+y^2-2ax-3a^2=0$ with respect to the circle $x^2+y^2+2ax-3a^2=0$ will be a tangent to the parabola $y^2=-4ax$. 5

7. Show that the area of the triangle formed by the straight lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is $\frac{\sqrt{h^2-ab}}{am^2-2hlm+bl^2}$. 5

8. Obtain the equation of the straight line passing through the point (2, 3, 5) and perpendicular to the intersection of the planes $x+2y-1=0$ and $2y+3z-5=0$. 5

Group – B
(Answer any five questions) [5×5]

9. a) Find the order and degree of $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{3/2}=xe^x$. 2

   b) Find the differential equation of the system of circles of constant radius a with their centres on the *x*-axis. 3

10. a) Define the exact differential equation of the first order and first degree. 2

b) Check whether $y(x^2+y^2+a^2)\frac{dy}{dx}+x(x^2+y^2-a^2)=0$ is an exact differential equation and hence solve it. 3

11. a) Define and give an example of second order homogeneous linear differential equation. 2

b) Solve: $xy^2dy=(x^3+y^3)dx$. 3

12. a) Find the integrating factor of $\frac{dy}{dx}+\frac{4x}{x^2+1}y=\frac{1}{(x^2+1)^3}$. 2

b) Solve: $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y$. 3

13. a) Define orthogonal trajectories. 2

b) Prove that the equation of the system of orthogonal trajectories of a series of confocal and co-axial parabolas $r=\frac{2a}{1+\cos\theta}$ is $r=\frac{2c}{1-\cos\theta}$. 3

14. a) Define and give examples of Euler and Bernouli differential equation. 2

b) Find the general solution of the Euler equation $t^2\frac{d^2y}{dt^2}-3t\frac{dy}{dt}+7y=0,\ t>0$. (Use the change of variable $x = \ln t$). 3

15. Obtain the complete primitive of the equation $y=px+\sqrt{a^2p^2+b^2}$ and also show that the singular solution is the ellipse which passes through the origin. 5

16. Solve by method of variation of parameters $\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\cos x$. 5

Group – C

(Answer any five questions) [5×5]

17. Let $T:\mathbb{R}^2\to\mathbb{R}^3$ be defined by $T(a_1,a_2)=(a_1-a_2,a_1,2a_1+a_2)$. Let β be the standard ordered basis for $\mathbb{R}^2$ and $\gamma=\{(1,1,0),(0,1,1),(2,2,3)\}$. Compute $[T]_\beta^\gamma$. If $\alpha=\{(1,2),(2,3)\}$, compute $[T]_\alpha^\gamma$. 5

18. a) Show that the union of two subspaces of a vector space may not be a subspace. (Give counter example) 2

b) Show that if V_1,V_2 be two subspaces of a vector space V, then $V_1+V_2=\{v_1+v_2 \mid v_1\in V_1,\ v_2\in V_2\}$ is a subspace of V. 3

19. a) Let V and W be vector spaces and let $T:V\to W$ be linear. Show that T is one to one if and only if $N(T)=\{\theta\}$, where θ is the additive identity of V. 4

b) Define, linear transformation. 1

20. Prove that the composition of two linear transformations T_1 and T_2 defined by $T_1\circ T_2(f)=T_1(T_2(f))$ is a linear transformation. 5

21. Show that, intersection of two subspaces of a vector space *V* is a subspace of *V*. 5

22. Determine whether $\{(1,-3,-2),(-3,1,3),(-2,10,2)\}$ forms a basis of $\mathbb{R}^3$ or not. 5

23. Determine whether the following set is a subspace of $\mathbb{R}^3$ under the usual addition and scalar multiplication defined on $\mathbb{R}^3$

$$\{(a_1,a_2,a_3)\in\mathbb{R} \mid 2a_1-7a_2+5a_3=0\}.$$ 5

24. Suppose $T:\mathbb{R}^2\to\mathbb{R}^2$ is linear such that $T(1,0)=(1,4)$ and $T(1,1)=(2,5)$. What is $T(2,3)$? Is *T* one-to-one? 5

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